Table 3 Input data for tow configuration

Tension at body, lb	247.44
Cable angle at body, deg	69.96
Tow speed, fps	10.00
Length of cable, ft	130.0
Depth of cable at tow ship, ft	0.0
Cable diameter, in.	1.0
Cable material	steel

tension in the cable at the tow ship attachment point is slightly increased. The latter effect indicates that ignoring hydrostatic pressure may lead to a nonconservative design.

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Similarity in the Modeling of Cable Twisting and Looping

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I. Introduction

ORDINARY armored cable generates a twist moment under tension. An assembly of armored and armorless cables therefore represents a twist moment and twist deformation problem as well as a tension problem. Under certain conditions involving moment and tension, a section of cable may relieve twist by looping. If the yield stress of the armor wires is exceeded by subsequent pulling of a looped section, a kink will result.

In the installation of communications cables in deep water, the twist properties of the coaxial cable are important.[1] Excessive twist deformation can produce cable failure. Kinks in cable make recovery difficult or impossible.

Model tests which duplicate the full-scale installation appear attractive for making comparisons of different laying schemes and determining the conditions under which looping and fouling are likely. In order for model tests to be valid, certain conditions must be satisfied by the model. These conditions are embodied in the dimensionless similarity parameters, which must have the same value in the model as in the full-scale operation.

In Ref. 2, the similarity parameters were worked out for cases in which twist, bending, looping, and kinking are not important. More precisely, the shear forces associated with twist and bending are small compared to the tension. Only cable weight and tangential and normal drag appear in the equilibrium equations. This is the case for most cable configuration problems.

The additional considerations lead to more terms in the equations defining the configurations, and hence to additional similarity parameters. The general problem is impossible to solve analytically, except for highly simplified cases[3] and impractical to model. As pointed out in Ref. 2, however, twist and bending ordinarily have a strictly localized effect on the cable tension and configuration. This suggests that the general problem is divided into two sequential problems. First, the cable tension and configuration would be worked out, either analytically, or by models using the parameters of Ref. 2. Then twist and bending effects would be superposed, modeling only the regions in which these effects are important.

II. The Equations of Equilibrium

The equations of equilibrium for a segment of cable ds, as shown in Fig. 1, include three force equations and three moment equations. These constitute the starting point of the "thin-rod" theory of elasticity.[4]

One possible way to specify the deformation of a bent and twisted rod is to make use of a moving coordinate system. First it is assumed that the rod is laid in an unbent and untwisted state. The z' axis is made coincident with the central axis of the rod. The x' and y' axes form a right-handed system and are aligned with the principal axes of the cross section of the rod. If the cross section has axial symmetry, as in most cables, the attitude of the x' axis is arbitrary.

The moving coordinate system x, y, z, is defined for each location s in the deformed rod as follows. The moving coordinate system translates and rotates in such a way that, at any time, the z axis is parallel to the cable axis, and the x and y coincide with what were the x' and y' axes in the undeformed rod. The angular velocity of the x, y, z, frame when the rod is traversed at unit speed has the components κ, κ' and τ along the x, y and z axes. Thus κ and κ' are bending rates per unit length and τ is the twist rate per unit length.

Let the components of force along the x,y,z axes be N,N' and T; N and N' are shear forces and T is the tension. Let the components of the moment vector be G,G' and H; G and G' are bending moments and H is the twist moment. Assume that the curvature and the twist can be related to the moments through the bending resistances A and B, and the twist resistance C, as follows:

$$G = A\kappa,\ G' = B\kappa',\ H = C\tau. \tag{1}$$

If the cross-section is axially symmetric, A and B are identical.

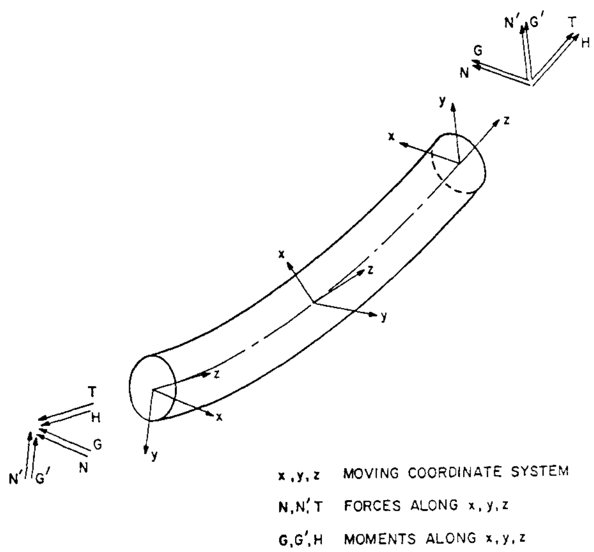

Fig. 1 Forces and moments on a segment of cable.

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The equations of equilibrium for the rod element ds are the force equations

$$\frac{dN}{ds} - N'\tau + T\kappa' + X = 0 \tag{2a}$$

$$\frac{dN'}{ds} + N\tau - T\kappa + Y = 0 \tag{2b}$$

$$\frac{dT}{ds} - N\kappa' + N'\kappa + Z = 0 \tag{2c}$$

and the moment equations

$$A\frac{d\kappa}{ds} - (B\text{-}C)\kappa'\tau - N' = 0 \tag{3a}$$

$$B\frac{d\kappa'}{ds} + (A-C)\kappa\tau + N = 0 \tag{3b}$$

$$C\frac{d\tau}{ds} - (A-B)\kappa\kappa' = 0 \tag{3c}$$

In Eqs. (2), X, Y, and Z are the body force components in the x,y,z directions. In Eqs. (3), body moments have been excluded. The solution of Eqs. (2) and (3), subject to the appropriate boundary conditions yields the forces N, N', T and the deformations κ, κ', τ of the rod as functions of length s.

With a few notable exceptions, cables are circular and have axial symmetry. For circular cable, from Eq. (3c), the twist rate τ is a constant, and the twist moment H is a constant, from one end to the other.

III. The Conditions for Similarity

The natural characteristic length for modeling sea installations is the depth of water. The characteristic force is the product of depth times a cable weight per unit length. These are used in Ref. 2, which deals with cable tensions and configurations for the perfectly flexible case.

The similarity conditions for the general looping and tension model cannot all simultaneously be met, for practical reasons. However, as pointed out above, the regions of a configuration that are looped, or about to loop, can be modeled separately. In such a bend-loop region, the shear forces N and N' are not negligible compared to the tension T.

A characteristic force P is defined as the value of the tension at some point in the cable adjacent to, but not in, the bend-loop region. From P, a characteristic length λ is defined

$$\lambda = \left(\frac{A}{P}\right)^{1/2}$$

The quantities P and λ do not depend on depth of water and are appropriate for modeling small regions because they are locally defined.

The nondimensional form of Eqs. (2) and (3) is easily found. The dimensionless forces, N/P, N'/P, and T/P and the dimensionless deformations $\lambda\kappa$, $\lambda\kappa'$, and $\lambda\tau$ depend upon the dimensionless body forces $\lambda X/P$, $\lambda Y/P$, and $\lambda Z/P$ and the relative stiffness B/A and C/A. The distance scale is s/λ. For circular cable, B/A is automatically modeled.

The tensions applied at the ends are scaled to P. The dimensionless applied twist moment is $H/P\lambda$. Instead of twist moment, the end twist rate $\lambda\tau$ may be specified, making $H/P\lambda$ dependent.

A model is valid if the dimensionless independent variables are each equal to their full-scale values. These are the conditions for similarity.

IV. The Possibility of Scaling Down

The possibility and feasibility of meeting the conditions of similarity will be considered for some cases of practical importance.

First, if the model cable is geometrically similar to the full-scale cable, and its components are made of the same materials, the conditions on B/A and C/A are satisfied, and

$$A \propto d^4$$

where d is the cable diameter. If model and full-scale are in air, the body forces are proportional to cross-sectional area

$$X, Y, Z \propto d^2$$

Therefore, in a cable coilability test, if the cable twist rate and the diameter of the cable storage tank are scaled according to the cable diameter, all conditions for similarity will be satisfied. A coiling test is easy if a geometrically similar model cable is available.

V. Conclusions

Model tests of cable twisting and looping offer one way of solving problems that arise in complicated cable and array installations. Ordinarily, twisting and looping are important over small regions of the suspension, and only such regions need to be modeled.

The conditions for similarity involve the bending and twisting resistances of the cables, the applied tensions and moments, and, in some cases, the cable weights.

Cable model tests appear practical and feasible if suitable model cables are available or can readily be made.

Acknowledgments

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